USF UNIVERSITY OF SOUTH FLORIDA

Statistical and Econometric Methods

Example: Poisson Regression – Likelihood Function Calculation

You are given data from six household and want to develop a model to study the number of cars owned by the household as a function of income (the only explanatory variable provided). The data are:

Household number	**Number of cars owned**	**Annual household income in thousands of dollars**
1	0	20
2	5	250
3	2	60
4	3	80
5	1	40
6	1	35

The model is estimated with only one explanatory variable (annual income, with no constant) and maximum likelihood results give $\beta = 0.0067$ (see estimation results on the next page). The likelihood function for a Poisson regression is: $L(\boldsymbol{\beta}) = \prod_i P(y_i)$ and with $\lambda_i = EXP(\boldsymbol{\beta}\mathbf{X}_i)$ applying the Poisson equation (Eq. 11.1 in the text) gives Eq. 11.3 (page 284 in the text);

$$P(y_i) = \frac{EXP\left[-EXP(\boldsymbol{\beta}\mathbf{X}_i)\right]\left[EXP(\boldsymbol{\beta}\mathbf{X}_i)\right]^{y_i}}{y_i!}; \qquad (11.1)$$

$$\text{so, } L(\boldsymbol{\beta}) = \prod_i \frac{EXP\left[-EXP(\boldsymbol{\beta}\mathbf{X}_i)\right]\left[EXP(\boldsymbol{\beta}\mathbf{X}_i)\right]^{y_i}}{y_i!} \qquad (11.3)$$

Assignment:

With the data provided, manually calculate the Likelihood function using Eq. 11.3. Then compute the log-likelihood and compare it to the Limdep output provided on the following page.

Variables available for your specification are (file TOBIT-LL-13.TXT)

Variable Number	Explanation
x1	Number of cars owned by the household
x2	Annual household income in thousands of dollars

Limdep output with TOBIT-LL-13.TXT data:

```
--> read;nvar=2;nobs=6;file=D:\old_drive_d\new_laptop\CE697N-disk\TOBIT-LL-13.TXT$
--> poisson;lhs=x1;rhs=x2$
+---------------------------------------------+
| Poisson Regression                          |
| Maximum Likelihood Estimates                |
| Model estimated: Aug 12, 2013 at 11:10:49AM.|
| Dependent variable                    X1    |
| Weighting variable                  None    |
| Number of observations                 6    |
| Iterations completed                   5    |
| Log likelihood function       -8.240650     |
| Number of parameters                   1    |
| Info. Criterion: AIC =          3.08022     |
|   Finite Sample: AIC =          3.24688     |
| Info. Criterion: BIC =          3.04551     |
| Info. Criterion:HQIC =          2.94128     |
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| Poisson Regression                          |
| Chi- squared =     2.43645  RsqP=   .6954   |
| G  - squared =     2.83718  RsqD=   .6784   |
| Overdispersion tests: g=mu(i)  : -1.537     |
| Overdispersion tests: g=mu(i)^2: -1.695     |
+---------------------------------------------+
+--------+-------------+----------------+--------+--------+----------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|
+--------+-------------+----------------+--------+--------+----------+
 X2      |     .00671260       .00167798     4.000   .0001   80.8333333
```