

# **Heterogeneity in Means and Variances**

## Traditional Random Parameters Models:

$$\beta_n = b + \varphi_n$$

We have some mean  $b$  and add an error term to account for possible heterogeneity (and assume a distribution for  $\varphi_n$ )

### Problem:

- We are not sure how accurately our chosen distribution of  $\varphi_n$  can track the unobserved heterogeneity
- What if the degree of unobserved heterogeneity is actually some function of explanatory variables?

## Heterogeneity in the means:

We allow the mean of the parameter to be a function of explanatory variables so:

$$\boldsymbol{\beta}_n = \boldsymbol{b} + \boldsymbol{\Theta}\boldsymbol{Z}_n + \boldsymbol{\varphi}_n$$

where:

- $\boldsymbol{b}$  is the mean parameter,
- $\boldsymbol{Z}_n$  is a vector of explanatory variables that influence the mean of  $\boldsymbol{\beta}_n$ ,
- $\boldsymbol{\Theta}$  is a vector of estimable parameters, and
- $\boldsymbol{\varphi}_n$  is a randomly distributed term that captures unobserved heterogeneity across crashes.

## **Potential Problem:**

- We still might be limited because we inherently assume a common variance (one standard deviation) across the parameters.

## **Possible Solution:**

- Allow the variance to be a function of explanatory variables as well.

## Heterogeneity in the means and variances:

We allow the mean of the parameter to be a function of explanatory variables so:

$$\boldsymbol{\beta}_{kn} = \mathbf{b} + \Theta_{kn} \mathbf{Z}_{kn} + \sigma_{kn} \text{EXP}(\boldsymbol{\omega}_{kn} \mathbf{W}_{kn}) \varphi_{kn}$$

where:

- $\mathbf{W}_{kn}$  is a vector of explanatory variables that captures heterogeneity in the standard deviation  $\sigma_{kn}$
- $\boldsymbol{\omega}_{kn}$  is the corresponding parameter vector, and
- All other terms as previously defined,

## Recent Papers:

Behnood, A., Mannering, F., 2017. The effect of passengers on driver-injury severities in single-vehicle crashes: A random parameters heterogeneity-in-means approach. *Analytic Methods in Accident Research* 14, 41-53.

Seraneeprakarn, P., et al., 2017. Occupant injury severities in hybrid-vehicle involved crashes: A random parameters approach with heterogeneity in means and variances. *Analytic Methods in Accident Research* 15, 41-55.

Behnood, A., Mannering, F., 2017. Determinants of bicyclist injury severities in bicycle-vehicle crashes: A random parameters approach with heterogeneity in means and variances. *Analytic Methods in Accident Research* 16, 35-47.