Pareto-Improving Congestion Pricing on General Transportation Networks

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Congestion Pricing

• The basic concept has been around for over 80 years (e.g., Pigou, 1920 and Knight, 1924)
  – Internalize the negative externality of a trip by charging up to the full cost that the trip imposes to the society
  – Use tolls to provide incentives for users to change their travel behaviors to reduce traffic congestion, enhance system performance and improve social benefits
Congestion Pricing Practice

- Cordon Pricing
  - Singapore, Oslo, London and Stockholm etc
- High-Occupancy/Toll Lanes
  - I-15 in California
  - I-394 in Minnesota
  - I-95 in Florida
  - Others
- U.S. Congress established Value Pricing Pilot Program
- The UK government has plans to introduce trial road-pricing schemes across England
Source: [www.stockholmsforsoket.se](http://www.stockholmsforsoket.se), the official webpage for The Stockholm Congestion Pricing Trial
Public Acceptance

• Tolling is unappealing to the public
  – Despite the success of the Stockholm Congestion Charging Trial in 2006, the results from the referendum are unclear
    • Stockholm: 53% For and 47% Against
    • 14 surrounding municipalities: 39.8% For and 60.2% Against.
  – The possible loss of welfare associated with, e.g., having to switch to less desirable routes, departure times or transportation modes because the more desirable ones are no longer affordable
Pareto-Improving Pricing Approach

- When compared to the situation without any pricing intervention, Pareto-improving tolls increase the social benefit without increasing travel-related expense of every stakeholder that may include individual road users, transit passengers, transit operators, transportation authorities, etc.

- Our premise: Pareto-improving schemes are more appealing to the public.

- Neither rationing (e.g., Daganzo, 1995) nor revenue distribution (e.g., Yang and Guo, 2005) is required to achieve a Pareto improvement.
Example

Cost function: $t_{ij}(v_{ij})$

- $t_{ij}(v_{ij}) = \text{travel time for arc } (i, j), \text{ where } v_{ij} \text{ is the flow on the arc}$
- There are 3.6 travelers for OD pair (1, 4)
Flow Distributions

- **User Equilibrium (UE) Distribution**
  - Every user is a utility maximizer
  - No user has any incentive to switch travel routes
  - Longer system delay

- **System Optimal (SO) Distribution**
  - Minimum total travel time or system delay
  - Some users have to use longer routes
Example (Cont’d)

- User equilibrium solution with total delay = 255.82.
  - Send 1.3223 units along path 1 – 3 – 4 with travel time 71.06.
  - Send 2.2772 units along path 1 – 3 – 2 – 4 with travel time 71.06.
  - Send 0 units along path 1 – 2 – 4 with travel time 72.78.
Example (Cont’d)

- **System optimal solution with total delay = 227.11.**
  - Send 1.1685 units along 1 – 3 – 4 with travel time 51.85.
  - Send 0.8956 units along 1 – 3 – 2 – 4 with travel time 55.85.
  - Send 1.5359 units along 1 – 2 – 4 with travel time 75.85.
Marginal Cost Pricing

- Set the toll on arc \((i, j)\) equal to its marginal external cost at the system optimum \(v^*\)

\[ \tau_{ij} = t'_{ij}(v^*)v^* \]

\[
\begin{align*}
\tau_{13} &= 10v_{13} + 20.641 \\
\tau_{32} &= 2 + 25v_{32} + 29.21 \\
\tau_{24} &= 10v_{24} + 24.32 \\
\end{align*}
\]
Marginal Cost Pricing: Example

- System optimum is in a tolled user equilibrium
  - Send 1.1685 units along 1 – 3 – 4 with time $51.85 + 49.85 = 101.70$
  - Send 0.8956 units along 1 – 3 – 2 – 4 with time $55.85 + 45.85 = 101.70$
  - Send 1.5359 units along 1 – 2 – 4 with time $75.85 + 25.85 = 101.70$

Tolled User Equilibrium Flows

Resulting Travel Times
Marginal Cost Pricing: Comparison

Without tolls:
Time of 1 – 3 – 4 = 71.06.
Time of 1 – 3 – 2 – 4 = 71.06.
Total Delay = 255.82

With MC tolls:
Cost of 1 – 3 – 4 = 101.70
Cost of 1 – 3 – 2 – 4 = 101.70
Cost of 1 – 2 – 4 = 101.70
Total Delay = 227.11
Toll Revenue = 139.02
Marginal Cost Pricing

• Marginal cost and other first-best pricing schemes are “most likely doomed to be political failures” because users will find themselves worse off (Hau, 2005).
Pareto-Improving Tolls

- Consider the following set of tolls
Pareto-Improving Tolls (Cont’d)

- Resulting Flow Distribution has a total delay of 241.17
  - Send 1.79 units along 1 – 3 – 4 with time 69.15 + 0.31 = 69.46
  - Send 0.45 units along 1 – 3 – 2 – 4 with time 50.95 + 18.51 = 69.46.
  - Send 1.36 units along 1 – 2 – 4 with time 69.46
Pareto-Improving Tolls: Comparison

Without tolls:
Time of 1 – 3 – 4 = 71.06.
Time of 1 – 3 – 2 – 4 = 71.06.
Total Delay = 255.82

With PI tolls:
Cost of 1 – 3 – 4 = 69.46
Cost of 1 – 3 – 2 – 4 = 69.46
Cost of 1 – 2 – 4 = 69.46
Total Delay = 241.17
Revenue = 8.88
Pareto-Improving Tolls

• Why is it possible?
• Is it always possible? If not, how do we know when it is?
• How do we find the tolls?
More on User Equilibrium

- User equilibrium is essentially a Nash equilibrium in Game Theory
  - Game theory is concerned with the general analysis of strategic interactions among multiple players
- Nash equilibrium in simultaneous non-cooperative games
  - Each player’s choice is optimal given others’ choice
  - No player would find it in his or her interest to deviate unilaterally from a Nash equilibrium strategy
Prisoner’s Dilemma

- Nash equilibrium does not necessarily lead to Pareto efficiency.

<table>
<thead>
<tr>
<th></th>
<th>Play B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Play A</td>
<td>Confess</td>
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<tr>
<td>Confess</td>
<td>-3, -3</td>
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<tr>
<td>Deny</td>
<td>-6, 0</td>
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## Dominating Flow Distribution

<table>
<thead>
<tr>
<th>Link</th>
<th>UE</th>
<th>DF-F-1</th>
<th>DF-F-2</th>
<th>DF-F-3</th>
<th>SO</th>
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<tbody>
<tr>
<td>(1, 3)</td>
<td>3.60</td>
<td>1.90</td>
<td>2.24</td>
<td>2.24</td>
<td>2.06</td>
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<tr>
<td>(1, 2)</td>
<td>0.00</td>
<td>1.70</td>
<td>1.36</td>
<td>1.36</td>
<td>1.54</td>
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<tr>
<td>(3, 2)</td>
<td>2.28</td>
<td>0</td>
<td>0.45</td>
<td>0.61</td>
<td>1.53</td>
</tr>
<tr>
<td>(3, 4)</td>
<td>1.32</td>
<td>1.90</td>
<td>1.79</td>
<td>1.63</td>
<td>1.17</td>
</tr>
<tr>
<td>(2, 5)</td>
<td>2.28</td>
<td>1.70</td>
<td>1.81</td>
<td>1.97</td>
<td>2.43</td>
</tr>
<tr>
<td>Cost of the Longest Utilized Path.</td>
<td>71.06</td>
<td>68.7</td>
<td>69.4</td>
<td>71.06</td>
<td>75.85</td>
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<tr>
<td>System Cost or Total Delay</td>
<td>255.78</td>
<td>247.1</td>
<td>241.17</td>
<td>234.99</td>
<td>227.11</td>
</tr>
</tbody>
</table>

Minimum Delay = 227.11
### Pareto-Improving Tolls

<table>
<thead>
<tr>
<th>Link</th>
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<th>Pareto-Improving</th>
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<tbody>
<tr>
<td></td>
<td>Flow</td>
<td>Time</td>
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<tr>
<td>(1, 3)</td>
<td>3.6000</td>
<td>36.00</td>
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<tr>
<td>(1, 2)</td>
<td>0.0000</td>
<td>50.00</td>
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<td>(3, 2)</td>
<td>2.2778</td>
<td>12.28</td>
</tr>
<tr>
<td>(3, 4)</td>
<td>1.3222</td>
<td>35.06</td>
</tr>
<tr>
<td>(2, 4)</td>
<td>2.2778</td>
<td>22.78</td>
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<tr>
<td>Path</td>
<td></td>
<td></td>
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<tr>
<td>1-3-4</td>
<td>1.3222</td>
<td>71.06</td>
</tr>
<tr>
<td>1-3-2-4</td>
<td>2.2778</td>
<td>71.06</td>
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<tr>
<td>1-2-4</td>
<td>0.0000</td>
<td>72.78</td>
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<tr>
<td>Total demand</td>
<td>3.6000</td>
<td>3.6000</td>
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<tr>
<td>Cost to users</td>
<td>255.80</td>
<td>235.01</td>
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<td>Toll Revenue</td>
<td>0</td>
<td>20.79</td>
</tr>
<tr>
<td>Total Delay</td>
<td>255.78</td>
<td>234.99</td>
</tr>
</tbody>
</table>
Summary

Why is it possible?

- User equilibrium may not be Pareto efficient and dominated by another flow distribution
- A pricing scheme may be developed to evolve the system to the dominating flow distribution

Is it always possible?

- No

How do we know whether it is possible and how to design a Pareto-improving pricing scheme?
Pareto-Improving Toll (PIT) Problem

\[
\begin{align*}
\text{min} & \quad t(\nu)^T \nu \\
\text{s.t.} & \quad \nu \in V^F \\
& \quad f_r^w \left( \sum_a \delta_{ar} (t_a (\nu_a) + \tau_a) - \lambda_w \right) = 0, \quad \forall w, r \in P^w \\
& \quad \sum_a \delta_{ar} (t_a (\nu_a) + \tau_a) \leq \lambda_w, \quad \forall w, r \in P^w \\
& \quad \lambda_w \leq c_{w}^{UE}, \quad \forall w \\
& \quad \tau_a \geq 0, \quad \forall a \\
\end{align*}
\]

\[V^F = \{ \nu : \nu_a = \sum_w \sum_{r \in P^w} \delta_{ar} f_r^w; \sum_{r \in P^w} f_r^w = d_w, f_r^w \geq 0 \}\]
Properties of the PIT Problem

- If \((v^*, \tau^*)\) solves the PIT problem and \(t(v^*)^Tv^* < t(v^{UE})^Tv^{UE}\), then \(\tau^*\) is a vector of Pareto-improving tolls.
  - If \(t(v^*)^Tv^* = t(v^{UE})^Tv^{UE}\), then no Pareto-improving toll exists.
- The PIT problem belongs to a class of optimization problems difficult to solve (mathematical programs with complementarity constraints)
  - It violates a standard regularity condition.
  - Commercial software of nonlinear optimization problems are ineffective.
Our Solution Approach

- Solve a sequence of relaxed nonlinear optimization problems
  - Commercial software
- Use concepts from manifold suboptimization to improve the relaxation.
- Our algorithm
  - Stops after a finite number of iterations.
  - Produce a “strongly regular” solution.
    - Local optimal solutions are strongly regular.
Extension: PIT-Elastic Demand

\[
\begin{align*}
\text{min} & \quad t(v)^T v - \sum_w \int_0^{d_w} D_w^{-1}(z) dz \\
\text{s.t.} & \quad v = \sum_w x^w \\
& \quad Ax^w - E^w d_w = 0, \quad \forall w, \\
& \quad t_{ij} (v_{ij}) + \beta_{ij} \geq (\rho^w_i - \rho^w_j), \quad \forall w, \text{ and } (i, j) \in L, \\
& \quad x_{ij}^w (t_{ij} (v_{ij}) + \beta_{ij} - (\rho^w_i - \rho^w_j)) = 0, \quad \forall w, \text{ and } (i, j) \in L, \\
& \quad \rho^w_{o(w)} - \rho^w_{d(w)} \leq D^{-1}_w (d_w), \quad \forall w, \\
& \quad D^{-1}_w (d_w) \leq c_{UE}^w \quad \forall w \\
& \quad x_{ij}^w \geq 0, \beta_{ij} \geq 0 \quad \forall w, \text{ and } (i, j) \in L,
\end{align*}
\]
Extension: PIT-Heterogeneous Users

MCPI-P: \[ \min \sum_k \beta^k t(u)^T v^k \]

s.t. \( v \in V^F \)
\[ f_{r}^{w,k}\left( \sum_a \delta_{ar} \left( \beta^k t_a (u_a) + \tau_a \right) - \lambda^{w,k} \right) = 0 \quad \forall w, k, \text{and } r \in P^w_k \]
\[ \sum_a \delta_{ar} \left( \beta^k t_a (u_a) + \tau_a \right) \geq \lambda^{w,k} \quad \forall w, k, \text{and } r \in P^w_k \]
\[ \lambda^{w,k} \leq c_{w,k}^{UE} \quad \forall w, k \]
\[ \tau \geq 0 \]
Extension: Approximate PIT

- Pareto-improving tolls may not exist or lead to a significant reduction in congestion.
- Tolls are approximately Pareto-improving if
  - Users are slightly worse-off
  - Congestion decreases
Extension: Approximate PIT

\[ \min \quad t(v)^T v \]

\[ s.t. \quad v \in V^F \]

\[ f_r^w \left( \sum_a \delta_{ar} (t_a(v_a) + \tau_a) - \lambda_w \right) = 0, \quad \forall w, r \in P^w \]

\[ \sum_a \delta_{ar} (t_a(v_a) + \tau_a) \leq \lambda_w \quad \forall w, r \in P^w \]

\[ \lambda_w \leq (1 + \alpha)c_{w}^{UE} \quad \forall w \]

\[ \tau_a \geq 0, \quad \forall a \]

where \( 0 < \alpha \leq 0.1 \)
Numerical Results

- Networks
  - Nine Node: 18 links, 9 nodes, 4 O-D pairs
  - Sioux Falls: 76 links, 24 nodes, 528 O-D pairs
  - Hull: 798 links, 501 nodes, 158 O-D pairs

- Fixed Demands

- BPR Travel Time Functions
Networks

Nine-node Network

Sioux Falls
## Results

<table>
<thead>
<tr>
<th></th>
<th>Nine-Node</th>
<th>Sioux Falls</th>
<th>Hull</th>
</tr>
</thead>
<tbody>
<tr>
<td>System Optimal Travel Delay (min.)</td>
<td>2253.92</td>
<td>3514.39</td>
<td>50542.64</td>
</tr>
<tr>
<td>User Equilibrium Travel Delay (min.)</td>
<td>2455.87</td>
<td>3654.46</td>
<td>51350.74</td>
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</tbody>
</table>

**Dominating flow induced by exact PIT tolls**

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<tr>
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<tbody>
<tr>
<td>Travel Delay (min.)</td>
<td>2455.28</td>
<td>3654.30</td>
<td>51337.33</td>
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<tr>
<td>Reduction (% of max.)</td>
<td>0.29%</td>
<td>0.12%</td>
<td>1.66%</td>
</tr>
</tbody>
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**Dominating flow induced by approximate PIT tolls with $\alpha = 0.05$**

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</thead>
<tbody>
<tr>
<td>Travel Delay (min)</td>
<td>2361.16</td>
<td>3620.00</td>
<td>51146.68</td>
</tr>
<tr>
<td>Reduction (% of max.)</td>
<td>46.90%</td>
<td>24.63%</td>
<td>25.25%</td>
</tr>
</tbody>
</table>

**Dominating flow induced by approximate PIT tolls with $\alpha = 0.10$**

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</thead>
<tbody>
<tr>
<td>Travel Delay (min.)</td>
<td>2361.16</td>
<td>3619.32</td>
<td>51086.23</td>
</tr>
<tr>
<td>Reduction (% of max.)</td>
<td>46.90%</td>
<td>25.09%</td>
<td>32.73%</td>
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</tbody>
</table>
Increase of O-D Travel Time (Sioux Falls)

Gini = 0.0541

Gini = 0.0176
Observations

- Pareto-improving tolls are relatively prevalent. However, as second-best pricing schemes, they may not lead to a significant improvement in the system performance.
- When Pareto-improving tolls do not exist or achieve the desired improvement, the approximate version may be able to do so without making users severely worse off as in the case with marginal cost pricing.
Multimodal Transportation Networks

- To further generalize the theory and achieve higher levels of improvement through the synergy among different modes, we further consider multimodal transportation networks that includes:
  - Transit services
  - High-occupancy/toll (HOT)
  - General-purpose or regular lanes
Pareto-Improving Tolls

• In this setting, a pricing scheme refers to a strategy for tolling roads and highways as well as adjusting fares on various transit lines.

• A Pareto improvement may be achieved through
  – better distributing travel demands among the available modes of transportation
  – revenue cross-subsidizing between transit and vehicle users
Pareto-Improving Tolls Problem

\[
\max \sum_w \frac{1}{\theta} \cdot \ln \left( \sum_m \exp(-\theta(\rho_{j,m}^w - \rho_i^{w,m} - \beta^m)) \cdot D^w - \sum_m \sum_l \tau_l^m \sum_w x_l^{w,m} \right)
\]

s.t.

\[
Ax^{w,m} = E^{w,m} d^{w,m}, \quad \forall w \in W, m \in M,
\]

\[
\sum_{w \in W} x_l^{w,T} \leq c_l^T, \quad \forall l \in L^T,
\]

\[
x_l^{w,T} \leq f_i \omega_i^w, \quad \forall i \in N, l \in L_i^+, w \in W,
\]

\[
\sum_{m \in M} d^{w,m} = D^w, \quad \forall w \in W,
\]

\[
x_l^{w,m} \geq 0, \quad \forall l \in L, w \in W, m \in M,
\]

\[
x_l^{w,T} = 0, \quad \forall l \in L^S \cup L^H, w \in W,
\]

\[
x_l^{w,m} = 0, \quad \forall l \in L^T, w \in W, m \in \{S, H\}.
\]
Pareto-Improving Tolls Problem (Cont’d)

\[(t_l(x) + \tau^m_l - (\rho^m_j - \rho^m_i))x^{w,m}_l = 0, \quad \forall l \in \{L^S \cup L^H\}, w \in W, m \in \{S, H\}, l \in L^+_i, l \in L^-_j, \]

\[(t_l(x) + \tau^T_l + \mu^w_l + \gamma_l - (\rho^T_j - \rho^T_i))x^{w,T}_l = 0, \quad \forall l \in L^T, w \in W, l \in L^+_i, l \in L^-_j, \]

\[\frac{1}{\theta}(\ln d^{w,m} + \beta^m) + \lambda^w - E^{w,m} \rho^{w,m} = 0, \quad \forall w \in W, m \in M, \]

\[\sum_{\forall l \in L^+_i} f_i \mu^w_l = 1, \quad \forall i \in I, w \in W, \]

\[\gamma_l(\sum_{w \in W} x^{w,T}_l - c^T_l) = 0, \quad \forall l \in L^T, \]

\[\mu^w_l (x^{w,T}_l - f_i \omega^w_l) = 0, \quad \forall l \in L^T, w \in W, \]

\[t_l(x) + \tau^m_l - (\rho^m_j - \rho^m_i) \geq 0, \quad \forall l \in L, w \in W, m \in \{S, H\}, l \in L^+_i, l \in L^-_j, \]

\[t_l(x) + \tau^T_l + \mu^w_l + \gamma_l - (\rho^T_j - \rho^T_i) \geq 0, \quad \forall l \in L, w \in W, l \in L^+_i, l \in L^-_j, \]

\[\gamma_l \geq 0, \quad \forall l \in L^T, \]

\[\mu^w_l \geq 0, \quad \forall l \in L^T, w \in W, \]
Pareto-Improving Tolls Problem (Cont’d)

\[ \tau_l^H = \tau_l^S, \quad \forall l \in L^S, \]
\[ \tau_l^S = 0, \quad \forall l \in L^T, \]
\[ \tau_l^H = 0, \quad \forall l \in \{L^H \cup L^T\}, \]
\[ \tau_l^T = 0, \quad \forall l \in \{L^S \cup L^H\}, \]
\[ \tau_l^H, \tau_l^S \geq 0, \quad \forall l \in L, \]
\[ -E^{w,m}_w \rho^{w,m}_w \leq t^{w,m}_{UE}, \quad \forall w \in W, m \in M, \]
\[ \sum_w \sum_m \sum_l \tau_l^m x_l^{w,m} \geq 0, \quad \forall w \in W, m \in M, l \in L. \]
Numerical Example

Nine-node Network

O-D demand:
- 1-3: 30
- 1-4: 30
- 2-3: 30
- 2-4: 40
## Results

<table>
<thead>
<tr>
<th></th>
<th>SO</th>
<th>UE</th>
<th>Pareto-improving</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social benefits</td>
<td>-1875.90</td>
<td>-2603.27</td>
<td>-2068.95</td>
</tr>
<tr>
<td>System travel time</td>
<td>1994.13</td>
<td>2851.63</td>
<td>2241.59</td>
</tr>
<tr>
<td>Revenue</td>
<td>513.97</td>
<td>0.00</td>
<td>70.22</td>
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<tr>
<td><strong>Demand</strong></td>
<td></td>
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<tr>
<td></td>
<td>SOV</td>
<td>HOV</td>
<td>Transit</td>
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<tr>
<td>1-3</td>
<td>7.79</td>
<td>13.56</td>
<td>8.65</td>
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<td>14.67</td>
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<td>10.95</td>
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<td>2-4</td>
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<td><strong>Travel cost</strong></td>
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<td>HOV</td>
<td>Transit</td>
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<td>23.22</td>
<td>19.41</td>
<td>20.02</td>
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<td>2-4</td>
<td>23.38</td>
<td>17.06</td>
<td>24.40</td>
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<td><strong>Travel cost increase (%)</strong></td>
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<td></td>
<td></td>
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<tr>
<td>1-3</td>
<td>9.8</td>
<td>-24.9</td>
<td>-44.6</td>
</tr>
<tr>
<td>1-4</td>
<td>4.5</td>
<td>-12.6</td>
<td>-33.9</td>
</tr>
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## Tolls

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Concluding Remarks

- Pareto-improving tolls
  - A new class of second-best tolls
- These tolls increase the social benefit without making any user worse off when compared to the situation without any pricing intervention. Additionally, the Pareto-improving charging scheme requires neither rationing nor revenue distribution, but relies on a fact that the original Wardropian user equilibrium flow distribution may not be Pareto efficient.
- Pareto-improving tolls are relatively prevalent. However, the tolls may not lead to a significant improvement in the system performance.
Concluding Remarks (Cont’d)

- Pareto improvement can be further enhanced on multimodal transportation networks by better distributing travel demands among the available modes of transportation and revenue cross-subsidizing between transit and vehicle users.

- The Pareto-improving toll problems can be formulated as mathematical programs with complementarity constraints, which can be solved effectively using a manifold suboptimization technique.
Acknowledgement

• This research is support in parts by grants from the National Science Foundation (CMMI-0653804) and the Center of Multimodal Solutions for Congestion Mitigation, University of Florida.