Location and Capacity Modeling of Multimodal Networks Interchanges

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Outline

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Problem Overview

An urban multimodal interchange is the place where it is possible to switch from one transportation mode to another.

An example of an interchange could be a station that communicates the street network (mode: Car) with a bus network (mode: Transit).
Motivation

- Need for improved transportation alternatives and infrastructure
- Increased demand
- Urban sprawl
- Fuel prices
- Reduction of emissions

Transit capital expenses in 2005 were 12.8 Billion, 26.3% corresponded to Bus projects (APTA, Public Transportation Fact book, May 2007)

Capital cost decisions:
- Land acquisition
- Number, size, and length of stations
- Number of tracks or lanes
- Number and size of parking lots or garages...
Intellectual Merit

The design of urban multimodal interchange involves the optimization of investments in infrastructure.

Mathematical modeling of these decisions have to take into account:

- Central authority objective
- Users objectives
- Capacity constraints
- Geographically referenced data
- Demand uncertainty
- Budget constraints

The resulting models are large scale mixed-integer non-linear problems requiring efficient solution.
Problem Description

Formulation and solution methodology for the problem of communicating two or more networks subject to user behavior and uncertain demand.

Application in transportation: Urban multimodal interchange design. Decision of intermediate locations where roadway users can switch to bus or train to continue to their final destination.

**Keywords:** Equilibrium, Traffic Assignment, Mode Choice, Stackelberg Games, Complementarity Problems, Variational Inequalities, MPEC, Discrete Network Design, Integer programming, Non-linear programming, Global Optimization
Problem Description (2)

Network Design Problem

Leader

- Topology, Capacity, Fares

System Optimum
- Budget, User Behavior, Demand Uncertainty

Stackelberg Game

- Departure time, Mode, Routes, Interchange

User Equilibrium

Followers

- Performance, Utilization

Network Attributes

Self benefit
- Origin-Destination pairs, Perceived cost, Socio-econ. Attributes

System Optimum
- Budget, User Behavior, Demand Uncertainty
Previous Research
Previous Research

Demand Uncertainty

Stoch. Inelastic
Determ.
Stoch. Elastic
Determ.
Stoch. Inelastic
Determ.
Stoch. Elastic
Determ.
Stoch. Inelastic
Determ.
Stoch. Elastic
Determ.

Demand Problem

Design

Disc.
Cont.
Unimodal
Dynamic
Traffic Assignment

Route Choice

Unimodal

Transportation Modes

Traffic Assignment

Multimodal

Transportation Modes

Demand Type

Stoch.
Determ.
Inelastic
Elastic

Transportation Modes

Demand Type

Stoch.
Determ.
Inelastic
Elastic

Transportation Modes

Demand Type

Stoch.
Determ.
Inelastic
Elastic

Transportation Modes
Selected Literature


Example (Adapted from Marin et. al. 2002)

**Notation:**

- **a**: Index for park and ride trips (car-transit)
- **b**: Index for transit-transit trips
- **c**: Other modes not using park and ride
- **W**: Set of O-D pairs \( \omega \)
- **g_\omega**: Demand for O-D pair
- **t**: Index for park and ride \( t \)
- **T_\omega**: Set of all the park and ride locations feasible for route \( \omega \)
Mathematical Model (1)

\[
\text{Min } \{ L(y) + R(y, z) + P(u) - B(g, v) \}
\]

- Location cost \( (L) \)
- Cost of providing transit service \( z \) to location \( y \)
- Cost related to the parking capacity \( u \)
- Performance measure as function of the demand \( g \) and fare \( v \)

Upper Level Problem \( \text{UPL}(g) \)

Design variables \( x \)

Location: \( y \in Y \subset \{0,1\}^m \)
Transit design: \( z \in Z \subset Z^m \)
Parking design: \( u \in U \subset \mathbb{R}^m \)
Parking Fare: \( v \in V \subset \mathbb{R}^m \)

Combination parking lots-transit: \( (y, z) \in S \subset \{0,1\}^m \times \mathbb{R}^m \)

\( m \) is the number of locations (existing + proposed) for the interchanges
Mathematical Model (2)

Upper Level Problem UPL\(g\)

\[
\min_{x \in X} ULP(x, g)
\]

Subject to:

Lower Level Problem LLP\(x\)

\[
g = \arg \min_{q \in \Omega(x)} LLP(q, x)
\]

Design variables \(x\)

Topology, Capacity, Fares

Leader

Performance, Utilization \(g\)

Network Attributes

Followers
Mathematical Model (3)

Demand modeling

\[
\left[ \bar{g}_\omega \right]_{\omega \in W}
\]

O-D pairs \( \omega \)

\begin{align*}
\text{car} & \quad \text{a} \\
\text{transit} & \quad \text{b} \\
\text{other} & \quad \text{c}
\end{align*}

Interchanges

Parking locations...

t : 1, 2, 3...

Mode

\[
G^k_\omega \left( U^k_\omega \right) = \frac{e^{-(\alpha^k + \beta_1 U^k_\omega)}}{\sum_{k' \in \{a, b, c\}} e^{-(\alpha^{k'} + \beta_1 U^{k'}_\omega)}}
\]

Interchange

\[
G^a_{\omega, t} \left( U^a_\omega \right) = \frac{e^{-(\alpha_t + \beta_2 U^a_{\omega, t})}}{\sum_{t' \in T_\omega} e^{-(\alpha_{t'} + \beta_2 U^a_{\omega, t'})}}
\]

Utility

\[
U^a_\omega = \frac{-1}{\beta_2} \log \left[ \sum_{t' \in T_\omega} e^{-(\alpha_{t'} + \beta_2 U^a_{\omega, t'})} \right]
\]

Demand

\[
g^a_{\omega, t} = G^a_{\omega, t} \left( U^a_\omega \right) \quad G^a_\omega (U_\omega) \bar{g}_\omega
\]
Mathematical Model (4)

Capacity modeling (1)

\[ c_t(g,u,v) = v_t + B_t \left( \frac{g_t}{u_t} \right)^n \]

- \( c_t(g,u,v) \): Fare at parking lot \( t \)
- \( B_t \): Capacity at parking lot \( t \)
- \( g_t \): Number of users in the parking lot
- \( u_t \): Car occupancy

\[ g_t = \sum_{\omega \in W_t} g_{\omega,t}^a \]

Capacity modeling (2)

\[ g_t \leq u_t \]
Mathematical Model (5)

$$\min_{g} LLP(g, x) = \sum_{\omega \in W} \left[ \sum_{t \in T_\omega} \left( \int_{0}^{g_t} c_t(s, x) ds + C_{\omega,t}^a g_{\omega,t}^a \right) + \sum_{k \in \{b,c\}} U^k_{\omega} g^k_{\omega} \right] + G(g)$$

$$\sum_{k \in \{a,b,c\}} g^k_{\omega} = \overline{g}_{\omega} \quad \text{Demand for OD pairs}$$

$$\sum_{t \in T_\omega(x)} g_{\omega,t}^a = g_{\omega}^a \quad \text{Demand by mode choice}$$

$$\sum_{\omega \in W_t} g_{\omega,t}^a = g_t \quad \text{Demand by interchange}$$
General Solution Strategy(1)

- Fixed-point problem
- Variational inequalities problems (VIP)
- Equilibrium problems

- Existing of solution by Brouwer’s fixed point theorem

Unconstrained nonlinear problems
- Newton’s Method
- BFGS
- Trust region methods

Constrained nonlinear problems
- Simplicial decomposition
- Penalty and barrier
- Fank-Wolfe

Reformulation as nonlinear programming problems
General Solution Strategy (2)

\[
\sum_{\omega \in W} \left[ \sum_{t \in T_\omega} \left( \int_0^l c_t(s, x) ds + C^a_{\omega, t} g^a_{\omega, t} \right) \right] + \sum_{k \in \{b, c\}} U^k_\omega g^k_\omega + G(g)
\]

\[
G(g) = \left( \frac{1}{\beta_1} \right) \sum_{k \in \{a, b, c\}} \sum_{w \in W} g^k_\omega (\ln g^k_\omega - 1 + \alpha^k) - \left( \frac{1}{\beta_2} \right) \sum_{\omega \in W} g^a_\omega (\ln g^a_\omega - 1)
\]

\[
+ \left( \frac{1}{\beta_2} \right) \sum_{w \in W} \sum_{t \in T_\omega} g^{a, y}_{\omega, t} (\ln g^{a, y}_{\omega, t} - 1 + \alpha_t) - \left( \frac{1}{\beta_3} \right) \sum_{\omega \in W} \sum_{t \in T_\omega} g^{a, t}_{\omega, t} (\ln g^{a, t}_{\omega, t} - 1)
\]

\[\beta_1 < \beta_2 < \beta_3\] Ensures the convexity of the objective function
Solution Approaches

The linear BPP is NP-Hard (Jeroslow, 1985), the checking of a BPP is also an NP-Hard problem (Vicente, Savard and Judice, 1994).

Numerical approaches

- Cost approximation algorithm and variations (Patriksson, 1999) which includes:
  - Stochastic programming (Patil, Ukkusuri, 2007)

Artificial intelligence approaches

- Simulated annealing (García, R. and Marín, A., 2001)
- Genetic algorithms (Dimitriou, L., Tsekeris, T., & Stathopoulos, A., 2008)

The problem structure will be studied to devise special solutions algorithms that can be compared with the solution of state-of-the-art solvers.
Current Research

**Detailed decision process:** include decisions on whether or not to make a trip, mode choice, parking choice

**Capacity modeling:** implicit or explicit demand

**Incorporate congestion effects:** Consider the interaction between the modes throughout the network

**Combined equilibrium:** consider passenger assignment model using hyperpath formulations

**System vs User optimum:** Evaluate the tradeoffs between these two objectives

**Incorporate uncertainty:** Create a scenario generation tool to account for variations in the demand
Computational Resources

The resulting mathematical models (medium size) will be coded in AMPL and tested using COIN-OR open source solvers through the NEOS website for benchmarking purposes.

http://www-neos.mcs.anl.gov/

http://www.coin-or.org/index.html
Expected Contributions

Extend the application of operations research in the transportation field by developing a series of models to address emerging issues in transportation planning.

Post new models for capacity expansion in multimodal network systems under uncertainty and subject to user behavior.

The outcome of this research can be implemented in a transportation planning software (or in GIS) helping to make well-informed decisions in transportation planning.

The modeling framework and solution algorithm can be used for topology, pricing and capacity planning of highly utilized networks subject to equilibrium constraints.